Chapter 11
Adjustment of GPS Surveys

11-1. General

Differential carrier phase GPS survey observations are adjusted no differently from conventional surveys. Each three-dimensional GPS baseline vector is treated as a separate distance observation and adjusted as part of a trilateration network. A variety of techniques may be used to adjust the observed GPS baselines to fit existing control. Since GPS survey networks often contain redundant observations, they are usually (but not always) adjusted by some type of rigorous least squares minimization technique. This chapter describes some of the methods used to perform horizontal GPS survey adjustments and provides guidance in evaluating the adequacy and accuracy of the adjustment results.

11-2. GPS Error Measurement Statistics

In order to understand the adjustment results of a GPS survey, some simple statistical terms should be understood.

a. Accuracy. Accuracy is how close a measurement or a group of measurements are in relation to a "true" or "known" value.

b. Precision. Precision is how close a group or sample of measurements are to each other. For example, a low standard deviation indicates high precision. It is important to understand that a survey or group of measurements can have a high precision but a low accuracy (i.e., measurements are close together but not close to the known or true value).

c. Standard deviation. The standard deviation is a range of how close the measured values are from the arithmetic average. A low standard deviation indicates that the observations or measurements are close together.

11-3. Adjustment Considerations

a. This chapter deals primarily with the adjustment of horizontal control established using GPS observations. Although vertical elevations are necessarily carried through the baseline reduction and adjustment process, the relative accuracy of these elevations is normally inadequate for engineering and construction purposes. Special techniques and constraints are necessary to determine approximate orthometric elevations from relative GPS observations, as was covered in Chapter 6.

b. The baseline reduction process (described in Chapter 10) directly provides the raw relative position coordinates which are used in a 3D GPS network adjustment. In addition, and depending on the manufacturer's software, each reduced baseline will contain various orientation parameters, covariance matrices, and cofactor and/or correlation statistics which may be used in weighing the final network adjustment. Most least squares adjustments use the accuracy or correlation statistics from the baseline reductions; however, other weighing methods may be used in a least squares or approximate adjustment.

c. The adjustment technique employed (and time devoted to it) must be commensurate with the intended accuracy of the survey, as defined by the project requirements. Care must be taken to prevent the adjustment process from becoming a project in itself.

d. There is no specific requirement that a rigorous least squares type of adjustment be performed on USACE surveys, whether conventional, GPS, or mixed observations. Traditional approximate adjustment methods may be used in lieu of least squares and will provide comparable practical accuracy results.

e. Commercial software packages designed for higher order geodetic densification surveys often contain a degree of statistical sophistication which is unnecessary for engineering survey control densification (i.e., Second-Order or less). For example, performing repeated chi-square statistical testing on observed data intended for 1:20,000 base mapping photogrammetric control may be academically precise but, from a practical engineering standpoint, is inappropriate. The distinction between geodetic surveying and engineering surveying must be fully considered when performing GPS survey adjustments and analyzing the results thereof.

f. Connections and adjustments to existing control networks, such as the NGRS, must not become independent projects. It is far more important to establish dense and accurate local project control than to consume resources tying into First-Order NGRS points miles from the project. Engineering, construction, and property/boundary referencing requires consistent local control with high relative accuracies; accurate connections/references to distant geodetic datums are of secondary importance. (Exceptions might involve projects in support of military operations.) The advent of GPS surveying technology has provided a cost-effective means of tying previously poorly connected USACE projects to the NGRS, and simultaneously transforming the project to the newly defined NAD 83. In performing (adjusting) these connections, care must be taken not to distort or warp long-established project construction/boundary reference points.

11-4. Survey Accuracy

a. General. The accuracy of a survey (whether performed using conventional or GPS methods) is a measure of the difference between observed values and the true values (coordinates, distance, angle, etc.). Since the true values are rarely known, only estimates of survey accuracy can be made. These estimates may be based on the internal observation closures, such as on a loop traverse, or connections with previously surveyed points assumed to have some degree of reliability. The latter case is typically a traverse (GPS or conventional) between two previously established points, either existing USACE project control or the published NGRS network.

(1) GPS internal accuracies are typically far superior to most previously established control networks (including the NAD 83 NGRS). Therefore, determining the accuracy of a GPS survey based on misclosures with external points is not always valid unless statistical accuracy estimates (i.e., station variance-covariance matrices, distance/azimuth relative accuracy estimates, etc.) from the external network's original adjustment are incorporated into the closure analysis for the new GPS work. Such refinements are usually unwarranted for most USACE work.

(2) Most survey specifications and standards (including USACE) classify accuracy as a function of the resultant relative accuracy between two usually adjacent points in a network. This resultant accuracy is estimated from the statistics in an adjustment, and is defined by the size of a 2D or 3D relative error ellipse formed between the two points. Relative distance, azimuth, or elevation accuracy specifications and classifications are derived from this model, and are expressed either in absolute values (e.g., ±1.2 cm or ±3.5 in.) or as ratios of the propagated standard errors to the overall length (e.g., 1:20,000).

b. Internal accuracy. A loop traverse originating and ending from a single point will have a misclosure when observations (i.e., EDM traverse angles/distances or GPS baseline vectors) are computed forward around the loop back to the starting point. The forward-computed misclosure provides an estimate of the relative or internal accuracy of the observations in the traverse loop, or more correctly, the internal precision of the survey. This is perhaps the simplest method of evaluating the adequacy of a survey. (These point misclosures, usually expressed as ratios, are not the same as relative distance accuracy measures.)

(1) Internal accuracy estimates made relative to a single fixed point are obtained when so-called free, unconstrained, or minimally constrained adjustments are performed. In the case of a single loop, no redundant observations (or alternate loops) back to the fixed point are available. When a series of GPS baseline loops (or network) are observed, then the various paths back to the single fixed point provide multiple position computations, allowing for a statistical analysis of the internal accuracy of not only the position closure but also the relative accuracies of the individual points in the network (including relative distance and azimuth accuracy estimates between these points). The magnitude of these internal relative accuracy estimates (on a free adjustment) determines the adequacy of the control for subsequent design, construction, and mapping work.

(2) Loop traverses are discouraged for most conventional surveys due to potential systematic distance or orientation errors which can be carried through the network undetected. FGCS classification standards for geodetic surveys do not allow traverses to start and terminate at a single point. Such procedures are unacceptable for incorporation into the NGRS network; however, due to many factors (primarily economic), loop traverses or open-ended spur lines are commonly employed in densifying project control for engineering and construction projects. Since such control is not intended for inclusion in the NGRS and usually covers limited project ranges, such practices have been acceptable. Such practices will also be acceptable for GPS surveys performed in support of similar engineering and construction activities.

c. External accuracy. The coordinates (and reference orientation) of the single fixed starting point will also have some degree of accuracy relative to the network in which it is located, such as the NGRS if it was established relative to that system/datum. This "external" accuracy (or inaccuracy) is carried forward in the traverse loop or network; however, any such external variance (if small) is generally not critical to engineering and construction. When a survey is conducted relative to two or more points on an existing reference network, such as USACE project control or the NGRS, misclosures with these fixed control points provide an estimate of the "absolute" accuracy of the survey. This analysis is usually obtained from a final adjustment, usually a fully constrained least squares minimization technique or by other recognized traverse adjustment methods (Transit, Compass, Crandall, etc.).

(1) This absolute accuracy estimate assumes that the fixed (existing) control is superior to the survey being performed, and that any position misclosures at connecting points are due to internal observational errors and not the existing control. This has always been a long-established and practical assumption and has considerable legal basis in property/boundary surveying. New work is rigidly adjusted to existing control regardless of known or unknown deficiencies in the fixed network.

(2) Since the relative positional accuracies of points on the NGRS are known from the NAD 83 readjustment, and GPS baseline vector accuracy estimates are obtained from the individual reductions, variations in misclosures in GPS surveys are not always due totally to errors in the GPS work. Forcing a GPS traverse/network to rigidly fit the existing (fixed) network usually results in a degradation of the internal accuracy of the GPS survey, as compared with a free (unconstrained) adjustment.

11-5. Internal versus External Accuracy

Classical geodetic surveying is largely concerned with absolute accuracy, or the best-fitting of intermediate surveys between points on a national network, such as the NGRS. Alternatively, in engineering and construction surveying, and to a major extent in boundary surveying, relative, or local, accuracies are more critical to the project at hand. Thus, the absolute NAD 27 or NAD 83 coordinates (in latitude and longitude) relative to the NGRS datum reference are of less importance; however, accurate relative coordinates over a given project reach (channel, construction site, levee section, etc.) are critical to design and construction.

a. For example, in establishing basic mapping and construction layout control for a military installation, developing a dense and accurate internal (or relative) control network is far more important than the values of these coordinates relative to the NGRS.

b. On flood control and river and harbor navigation projects, defining channel points must be accurately referenced to nearby shore-based control points. These points, in turn, directly reference boundary/right-of-way points and are also used for dredge/construction control. Absolute coordinates (NGRS/NAD) of these construction and/or boundary reference points are of less importance.

c. Surveys performed with GPS, and final adjustments thereof, should be configured/designed to establish accurate relative (local) project control; of secondary importance is connection with NGRS networks.

d. Although reference connections with the NGRS are desirable and recommended, and should be made where feasible and practicable, it is critical that such connections (and subsequent adjustments thereto) do not distort the internal (relative) accuracy of intermediate points from which design, construction, and/or project boundaries are referenced.

e. Connections and adjustments to distant networks (i.e., NGRS) can result in mixed datums within a project area, especially if not all existing project control has been tied in. This in turn can lead to errors and contract disputes during both design and construction. On existing projects with long-established reference control, connections and adjustments to outside reference datums/networks should be performed with caution. The impacts on legal property and project alignment definitions must also be considered prior to such connections. (See also paragraph 8-3*d*.)

f. On newly authorized projects, or on projects where existing project control has been largely destroyed, reconnection with the NGRS is highly recommended. This will ensure that future work will be supported by a reliable and consistent basic network, while minimizing errors associated with mixed datums.

11-6. Internal and External Adjustments

GPS-performed surveys are usually adjusted and analyzed relative to their internal consistency and external fit with existing control. The internal consistency adjustment (i.e., free or minimally constrained adjustment) is important from a contract compliance standpoint. A contractor's performance should be evaluated relative to this adjustment. The final, or constrained, adjustment fits the GPS survey to the existing network. This is not always easily accomplished since existing networks often have lower relative accuracies than the GPS observations being fit. Evaluation of a survey's adequacy should not be based solely on the results of a constrained adjustment.

11-7. Internal or Geometric Adjustment

This adjustment is made to determine how well the baseline observations fit or internally close within themselves.

(Other EDM distances or angles may also be included in the adjustment.) It is referred to as a free adjustment. This adjustment provides a measure of the internal precision of the survey.

a. In a simplified example, a conventional EDM traverse which is looped back to the starting point will misclose in both azimuth and position, as shown in Figure 11-1. Classical "approximate" adjustment techniques (e.g., Transit, Compass, Bowditch, Crandall) will typically assess the azimuth misclosure, proportionately adjust the azimuth misclosure (usually evenly per station), recompute the traverse with the adjusted azimuths, and obtain a position misclosure. This position misclosure (in X and Y) is then distributed among all the points on the traverse using various weighing methods (distance, latitudes, departures, etc.). Final adjusted azimuths and distances are then computed from grid inverses between the adjusted points. The adequacy/accuracy of such a traverse is evaluated based on the azimuth misclosure and position misclosure after azimuth adjustment (usually expressed as a ratio to the overall length of the traverse).

b. A least squares adjustment of the same conventional loop traverse will end up adjusting the points similarly to the approximate methods traditionally employed. The only difference is that a least squares adjustment simultaneously adjusts both observed angles (or directions) and distance measurements. A least squares adjustment also allows variable weighting to be set for individual angle/distance observations, which is a somewhat more complex process when approximate adjustments are performed. In addition, a least squares adjustment will yield more definitive statistical results of the internal accuracies of each observation and/or point, rather than just the final closure. This includes estimates of the accuracies of individual station X-Y coordinates, relative azimuth accuracies, and relative distance accuracies.

c. A series of GPS baselines forming a loop off a single point can be adjusted and assessed similarly to a conventional EDM traverse loop described in *a* above (see Figure 11-1). The baseline vector components may be computed (accumulated) around the loop with a resultant 3D misclosure back at the starting point. These misclosures (in X, Y, and Z) may be adjusted using either approximate or least squares methods. The method by which the misclosure is distributed among the intermediate points in the traverse is a function of the adjustment weighting technique.

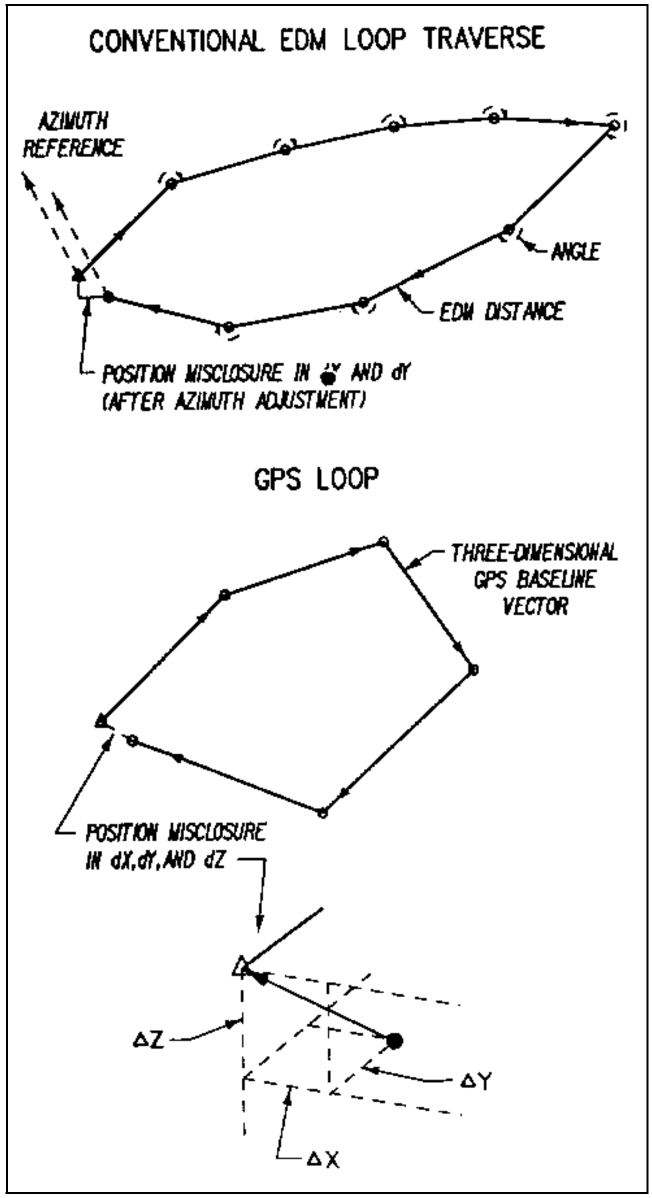

Figure 11-1. Conventional EDM and GPS traverse loops

(1) In the case of a simple EDM traverse adjustment, the observed distances (or position corrections) are weighted as a function of the segment length and the overall traverse length (Compass Rule), or to the overall sum of the latitudes/departures (Transit Rule). Two-dimensional EDM distance observations are not dependent on their direction; that is, a distance's X- and Y-components are uncorrelated.

(2) GPS baseline vector components (in X, Y, and Z) are correlated due to the geometry of the satellite solution; that is, the direction of the baseline vector is significant. Since the satellite geometry is continuously changing, remeasured baselines will have different correlations between the vector components. Such data are passed down from the baseline reduction software for use in the adjustment.

d. The magnitude of the misclosure (i.e., loop closure) of the GPS baseline vectors at the initial point provides an estimate of the internal precision or geometric consistency of the loop (survey). When this misclosure is divided by the overall length of the baselines, an internal relative accuracy estimate results. This misclosure ratio should not be less than the relative distance accuracy classification intended for the survey, per Table 8-1.

(1) For example, if the position misclosure of a GPS loop is 0.08 m and the length of the loop is 8,000 m, then the loop closure is 0.08/8,000 or 1 part in 100,000 (1:100,000).

(2) When an adjustment is performed, the individual corrections/adjustments made to each baseline (so-called residual errors) provide an accuracy assessment for each baseline segment. A least squares adjustment can additionally provide relative distance accuracy estimates for each line, based on standard error propagation between adjusted points. This relative distance accuracy estimate is most critical to USACE engineering and construction work and represents the primary basis for assessing the acceptability of a survey.

11-8. External or Fully Constrained Adjustment

The internal "free" geometric adjustment provides adjusted positions relative to a single, often arbitrary, fixed point. Most surveys (conventional or GPS) are connected between existing stations on some predefined reference network or datum. These fixed stations may be existing project control points (on NAD 27--SPCS 27) or stations on the NGRS (NAD 83). In OCONUS locales, other local or regional reference systems may be used. A constrained adjustment is the process used to best fit the survey observations to the established reference system.

a. A simple conventional EDM traverse (Figure 11-2) between two fixed stations best illustrates the process by which comparable GPS baseline vectors are adjusted. As with the loop traverse described in paragraph 10-8, the misclosure in azimuth and position between the two fixed end points may be adjusted by any type of approximate or least squares adjustment method. Unlike a loop traverse, however, the azimuth and position misclosures are not wholly dependent on the internal errors in the traverse--the fixed points and their azimuth references are not absolute, but contain relative inaccuracies with respect to one another.

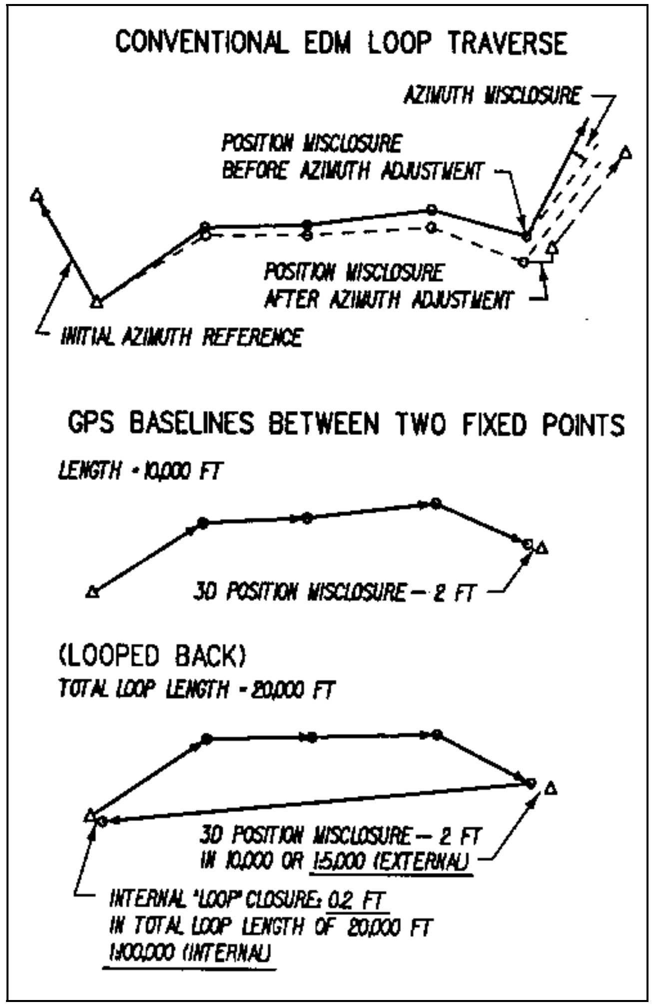

Figure 11-2. Constrained adjustment between two fixed points

b. A GPS survey between the same two fixed points also contains a 3D position misclosure. Due to positional uncertainties in the two fixed network points, this misclosure may (and usually does) far exceed the internal accuracy of the raw GPS observations. As with a conventional EDM traverse, the 3D misclosures may be approximately adjusted by proportionately distributing them over the intermediate points. A least squares adjustment will also accomplish the same thing.

c. If the GPS survey is looped back to the initial point, the free adjustment misclosure at the initial point may be compared with the apparent position misclosure with the other fixed point. In Figure 11-2, the free adjustment loop misclosure is 1:100,000 whereas the misclosure relative to the two network control points is only 1:5,000. Thus, the internal relative accuracy of the GPS survey is on the order of 1 part in 100,000 (based on the misclosure); if the GPS baseline observations are constrained to fit the existing control, the 0.6-m external misclosure must be distributed among the individual baselines to force a fit between the two end points.

(1) After a constrained adjustment, the absolute position misclosure of 0.6 m causes the relative distance accuracies between individual points to degrade. They will be somewhat better than 1:5,000 but far less than 1:100,000. The statistical results from a constrained least squares adjustment will provide estimates of the relative accuracies between individual points on the traverse.

(2) This example also illustrates the advantages of measuring the baseline between fixed network points when performing GPS surveys, especially when weak control is suspected (as in this example).

(3) Also illustrated is the need for making additional ties to the existing network. In this example, one of the two fixed network points may have been poorly controlled when it was originally established, or the two points may have been established from independent networks (i.e., were never connected). A third or even fourth fixed point would be beneficial in resolving such a case.

d. If the intent of the survey shown in Figure 11-2 was to establish 1:20,000 relative accuracy control, connecting between these two points obviously will not provide that accuracy given the amount of adjustment that must be applied to force a fit. For example, if one of the individual baseline vectors was measured at 600 m and the constrained adjustment applied a 0.09-m correction in this sector, the relative accuracy of this segment would be roughly 1:6,666. This distortion would not be acceptable for subsequent design/construction work performed in this area.

e. Most GPS survey networks are more complex than the simple traverse example in Figure 11-2. They may consist of multiple loops and may connect with any number of control points on the existing network. In addition, conventional EDM, angles, and differential leveling measurements may be included with the GPS baselines, resulting in a complex network with many adjustment conditions.

11-9. Partially Constrained Adjustments

In the previous example of the simple GPS traverse, holding the two network points rigidly fixed caused an adverse degradation in the GPS survey, based on the differences between the free (loop) adjustment and the fully constrained adjustment. Another alternative is to perform a semiconstrained (or partially constrained) adjustment of the net. In a partially constrained adjustment, the two network points are not rigidly fixed but only partially fixed in position. The degree to which the existing network points are constrained may be based on their estimated relative accuracies or, if available, their original adjustment positional accuracies (covariance matrices). Partially constrained adjustments are not practicable using approximate adjustment techniques; only least squares will suffice.

a. For example, if the relative distance accuracy between the two fixed network points in Figure 11-2 is approximately 1:10,000, this can be equated to a positional uncertainty between them. Depending on the type and capabilities of the least squares adjustment software, the higher accuracy GPS baseline observations can be best fit between the two end points such that the end points of the GPS network are not rigidly constrained to the original and two control points but will end up falling near them.

b. Adjustment software will allow relative weighting of the fixed points to provide a partially constrained adjustment. Any number of fixed points can be connected to, and these points may be given partial constraints in the adjustment.

c. Performing partially constrained adjustments (as opposed to a fully constrained adjustment) takes advantage of the inherent higher accuracy GPS data relative to the existing network control, which is traditionally weak on many USACE project areas. Less warping of the GPS data (due to poor existing networks) will then occur.

d. A partial constraint also lessens the need for performing numerous trial-and-error constrained adjustments in attempts to locate poor external control points causing high residuals. Fewer ties to the existing network need be made if the purpose of such ties was to find a best fit on a fully constrained adjustment.

e. When connections are made to the NAD 83, relative accuracy estimates of NGRS stations can be obtained from the NGS. Depending on the type of adjustment software used, these partial constraints may be in the form of variance-covariance matrices, error ellipses, or circular accuracy estimates.

11-10. Approximate Adjustments of GPS Networks

Simply constructed GPS networks used for establishing lower order (i.e., Second-Order and lower) USACE control can be effectively adjusted using approximate adjustment techniques, or adjustments which approximate the more rigorous least squares solution. Although least squares solutions may be theoretically superior to approximate methods, the resultant differences between the adjustments are generally not significant from a practical engineering standpoint.

a. Given the high cost of commercial geodetic adjustment software, coupled with the adjustment complexity of these packages, approximate adjustment methods are allowed for in-house and contracted surveys.

b. In practice, any complex GPS survey network may be adjusted by approximate methods. If the main loop/line closures are good, redundant ties to other fixed network points may be used as checks rather than being rigidly adjusted.

c. In some cases it is not cost-effective to perform detailed and time-consuming least squares adjustments on GPS project control surveys requiring only 1:5,000 or 1:10,000 engineering/construction/boundary location accuracy. If internal loop closures are averaging over 1:200,000, then selecting any simple series of connecting baselines for an approximate adjustment will yield adequate resultant positional and relative distance accuracies for the given project requirements. If a given loop/baseline series of say five points miscloses by 0.01 ft over 1,000 m (1:100,000), a case can be made for not even making any adjustment if a relative accuracy of only 1:5,000 is required between points.

d. Any recognized approximate adjustment method may be used to distribute baseline vector misclosures. The method used will depend on the magnitude of the misclosure to be adjusted and the desired accuracy of the survey. These include the following:

(1) Simple proportionate distribution of loop/line position misclosures among the new station coordinates.

(2) Compass Rule.

(3) Transit Rule.

(4) Crandall Method.

(5) No adjustment. Use raw observations if misclosures are negligible.

e. Approximate adjustments are performed using the 3D earth-centered X-Y-Z coordinates. The X-Y-Z coordinates for the fixed points are computed using the transform algorithms shown in *f* below or obtained from the baseline reduction software. Coordinates of intermediate stations are determined by using the baseline vector component differences (ΔX, ΔY, ΔZ) which are obtained directly from the baseline reductions. These differences are then accumulated (summed) forward around a loop or traverse connection, resulting in 3D position coordinate misclosures at the loop nodes and/or tie points. These misclosures are then adjusted by any of the methods in *d* above. GPS vector weighting is accomplished within the particular adjustment method used; there is no need to incorporate the standard errors from the baseline reductions into the adjustment. Internal survey adequacy and acceptance are performed based on the relative closure ratios, as in conventional traversing criteria (see FGCC 1984). Final local datum coordinates are then transformed back from the X-Y-Z coordinates.

f. Given a loop of baseline vectors between two fixed points (or one point looped back on itself), the following algorithms may be used to adjust the observed baseline vector components and compute the adjusted station geocentric coordinates.

(1) Given: Observed baseline vector components ΔX_i, ΔY_i, ΔZ_i for each baseline i (total of n baselines in the loop/traverse). The 3D length of each baseline is l_i, and the total length of the loop/traverse is L.

(2) The misclosures (dx, dy, and dz) in all three coordinates are computed from:

$$
\begin{aligned}
dx &= X_F + \sum_{i=1}^{i=n} \Delta X_i - X_E \\
dy &= Y_F + \sum_{i=1}^{i=n} \Delta Y_i - Y_E \\
dz &= Z_F + \sum_{i=1}^{i=n} \Delta Z_i - Z_E
\end{aligned}
\tag{11-1}
$$

Where X_F, Y_F, and Z_F are the fixed coordinates of the starting point and X_E, Y_E, and Z_E are the coordinates of the end point of the loop/traverse. (These misclosures would also be used to assess the internal accuracy of the work.)

(3) Adjustments (δx_i, δy_i, δz_i) to each baseline vector component may be computed using either the Compass Rule:

$$\delta x_i = - dx \left(\frac{l_i}{L} \right)$$
$$\delta y_i = - dy \left(\frac{l_i}{L} \right) \qquad (11\text{-}2)$$
$$\delta z_i = - dz \left(\frac{l_i}{L} \right)$$

or the Transit Rule:

$$\delta x_i = - dx \left(\frac{\Delta X_i}{\sum \Delta X_i} \right)$$
$$\delta y_i = - dy \left(\frac{\Delta Y_i}{\sum \Delta Y_i} \right) \qquad (11\text{-}3)$$
$$\delta z_i = - dz \left(\frac{\Delta Z_i}{\sum \Delta Z_i} \right)$$

(4) The adjusted vector components are computed from:

$$\Delta X_i^{\,a} = \Delta X_i + \delta x_i$$
$$\Delta Y_i^{\,a} = \Delta Y_i + \delta y_i \qquad (11\text{-}4)$$
$$\Delta Z_i^{\,a} = \Delta Z_i + \delta z_i$$

(5) The final geocentric coordinates are then computed by summing the adjusted vector components from Equation 11-4 above:

$$X_i^{\,a} = X_F + \Sigma \Delta X_i^{\,a}$$
$$Y_i^{\,a} = Y_F + \Sigma \Delta Y_i^{\,a} \qquad (11\text{-}5)$$
$$Z_i^{\,a} = Z_F + \Sigma \Delta Z_i^{\,a}$$

g. Example of an approximate GPS survey adjustment:

(1) Fixed control points from the U.S. Army Yuma Proving Ground GPS Survey (May 1990) (see Figure 11-3):

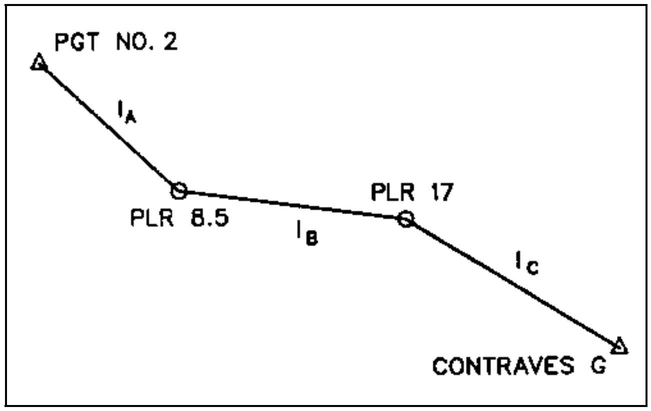

Figure 11-3. Yuma GPS traverse sketch

PGT NO 2:

X_F = (-) 2205 949.0762

Y_F = (-) 4884 126.7921

Z_F = + 3447 135.1550

CONTRAVES G:

X_E = (-) 2188 424.3707

Y_E = (-)4897 740.6844

Z_E = + 3438 952.8159

(XYZ geocentric coordinates were computed from GP-XYZ transform using Equations 11-6 and 11-7 below)

l_a, l_b, l_c = observed GPS baseline vectors (from baseline reductions)

and PLR 8.5 and PLR 17 are the points to be adjusted.

(2) Misclosures in X, Y, and Z (from Equation 11-1):

(-)2205	949.0762 X_F	(-)4884	126.7921 Y_F
+3	777.9104 ΔX_a	(-)6	006.8201 ΔY_a
+7	859.4707 ΔX_b	(-)3	319.1092 ΔY_b
+5	886.8716 ΔX_c	(-)4	288.9638 ΔY_c
-(-)2188	424.3707 X_E	-(-)4897	740.6844 Y_E

$dx = (-)\ 0.4528$ $dy = (-)\ 1.0008$

3447	135.1550 Z_F
(-)6	231.5468 ΔZ_a
+	400.1902 ΔZ_b
(-)2	350.2230 ΔZ_c
- 3438	952.8159 Z_E

$dz = +\ 0.7595$

(3) Linear 3D misclosure:

$= (0.4528^2 + 1.0008^2 + 0.7595^2)^{1/2} = \underline{1.335\text{ m}}$

or 1 part in 25,638.2/1.335 = 1:19,200

(Note: This is a constrained misclosure check, not free)

(4) Compass Rule adjustment:

(a) Compass Rule misclosure distribution:

$l_a = 9{,}443.869$ $l_a/L = 0.368$
$l_b = 8{,}540.955$ $l_b/L = 0.333$
$l_c = 7{,}653.366$ $l_c/L = 0.299$

$L = 25{,}638.190$ $\Sigma = 1.000$

(b) Compass Rule adjustment to GPS vector components using Equation 11-2:

Vector	δ_x	δ_y	δ_z
A	0.1666	0.3683	(-) 0.2795
B	0.1508	0.3333	(-) 0.2529
C	0.1354	0.2992	(-) 0.2271
	(+0.4528)	(+1.0008)	((-)0.7595) Check

(c) Adjusted baseline vectors (Equation 11-4):

Vector	ΔX^a	ΔY^a	ΔZ^a
A	3778.0770	(-)6006.4518	(-)6231.8263
B	7859.6215	(-)3318.7759	399.9373
C	5887.0070	(-)4288.6646	(-)2350.4501

(d) Final adjusted coordinates (Equation 11-5):

	X^a	Y^a
PGT No. 2	(-)2205 949.0762	(-)4884 126.7921
PLR 8.5	(-)2202 170.9992	(-)4890 133.2439
PLR 17	(-)2194 311.3777	(-)4893 452.0198
Contraves G (Check)	(-)2188 424.3707	(-)4897 740.6844

	Z^a	
PGT No. 2	+3447	135.1550
PLR 8.5	+3440	903.3287
PLR 17	+3441	303.2660
Contraves G (Check)	+3438	952.8159

(e) Adjusted geocentric coordinates are transformed to ϕ, λ, h, using Equations 11-9 through 11-13. Geographic coordinates may then be converted to local SPCS (either NAD 83 or NAD 27) project control using USACE program CORPSCON.

(5) Transit Rule adjustment.

(a) Distribution of GPS vector misclosures using Equation 11-3:

$$\Sigma \Delta X_i = 3777.9104 + 7859.4707 + 5886.8716 = 17{,}524.2527$$

Similarly,

$$\Sigma \Delta Y_i = 13{,}614.8931$$

$$\Sigma \Delta Z_i = 8{,}981.9600$$

$$\delta x_i = -dx \left(\frac{\Delta X_i}{\sum \Delta X_i} \right) = -(-) \frac{0.4528}{17{,}524.2527} \Delta X_i$$

$$= +2.584 \times 10^5 \quad \Delta X_i$$

Similarly,

$$\delta y_i = +7.351 \times 10^5 \; \Delta Y_i$$
$$\delta z_i = (-)8.456 \times 10^5 \; \Delta Z_i$$

(b) Adjustments to baseline vector components using Transit Rule (Equation 11-3):

Vector	δx	δy	δz
A	0.0976	0.4415	(-)0.5269
B	0.2031	0.2440	(-)0.0338
C	0.1521	0.3153	(-)0.1987
(check)	(0.4528)	(1.0008)	(- 0.7595)

(c) Adjusted baseline vectors (Equation 11-4):

Vector	ΔX^a	ΔY^a	ΔZ^a
A	3 778.0080	(-)6 006.3786	(-)6 232.0737
B	7 859.6738	(-)3 318.8652	+ 400.1564
C	5 887.0237	(-)4 288.6485	(-)2 350.4217

(d) Final adjusted coordinates (Equation 11-5):

	X^a	Y^a
PGT No. 2	(-)2 205 949.0762	(-)4884 126.7921
PLR 8.5	(-)2 202 171.0682	(-)4890 133.1707
PLR 17	(-)2 194 311.3944	(-)4893 452.0359
Contraves G (Check)	(-)2 188 424.3707	(-)4897 740.6844

	Z^a
PGT No. 2	+3447 135.1550
PLR 8.5	+3440 903.0813
PLR 17	+3441 303.2377
Contraves G (Check)	+3438 952.8160

(6) Proportionate distribution adjustment method.

(a) Vector misclosures are simply distributed proportionately over each of the three GPS baselines in the traverse:

$$\delta x = -(-)\frac{0.4528}{3} = +0.1509$$

$$\delta y = -(-)\frac{1.0008}{3} = +0.3336$$

$$\delta z = -(-)\frac{0.7595}{3} = (-)0.2532$$

Vector	ΔX^a	ΔY^a	ΔZ^a
A	3778.0613	(-) 6006.4865	(-) 6231.8000
B	7859.6216	(-) 3318.7756	+ 399.9370
C	5887.0225	(-) 4288.6302	(-) 2350.4762

(b) Final adjusted coordinates:

	X^a	Y^a
PLR 8.5	(-)2202 171.0149	(-)4890 133.2786
PLR 17	(-)2194 311.3933	(-)4893 452.0542

	Z^a
PLR 8.5	+3440 903.3550
PLR 17	+3441 303.2920

Note: Relatively large horizontal (2D) misclosure (1:23,340) may be due to existing control inadequacies, not poor GPS baseline observations.

(c) Variance between adjusted coordinates yields relative accuracies well in excess of 1:20,000; thus, if project control requirements are only 1:10,000, then any of the three adjustment methods may be used.

The recommended method is the Compass Rule.

Fixed coordinates of PGT No. 2 and CONTRAVES G can be on any reference ellipsoid -- NAD 27 or NAD 83.

11-11. Geocentric Coordinate Conversions

The following algorithms for transforming between geocentric and geographic coordinates can be performed in the field on a Hewlett-Packard-style hand-held calculator.

a. Geodetic to Cartesian coordinate conversion. Given geodetic coordinates on NAD 83 (in ϕ, λ, H) or NAD 27, the geocentric Cartesian coordinates (X, Y, and Z) on the WGS 84, GRS 80, or Clarke 1866 ellipsoid are converted directly by the following formulas.

$$X = (R_N + h)\cos\phi\cos\lambda$$
$$Y = (R_N + h)\cos\phi\sin\lambda \tag{11-6}$$
$$Z = \left(\frac{b^2}{a^2}R_N + h\right)\sin\phi$$

where

ϕ = latitude

λ = 360° - λ_W (for CONUS west longitudes)

h = the ellipsoidal elevation. If only the orthometric elevation H is known, then that value may be used.

The normal radius of curvature R_N can be computed from either of the following equations:

$$R_N = \frac{a^2}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}} \tag{11-7}$$

$$R_N = \frac{a}{\sqrt{1 - e^2\sin^2\phi}} \tag{11-8}$$

and

a (GRS 80) = 6,378,137.0 m (semimajor axis)
a (WGS 84) = 6,378,137.0 m
a (NAD 27) = 6,378,206.4 m

b (GRS 80) = 6,356,752.314 1403 m (semiminor axis)
b (WGS 84) = 6,356,752.314 m
b (NAD 27) = 6,356,583.8 m

f (GRS 80) = 1/298.257 222 100 88 (flattening)
f (WGS 84) = 1/298.257 223 563
f (NAD 27) = 1/294.978 698

e^2 (GRS 80) = 0.006 694 380 222 90 (eccentricity squared)
e^2 (WGS 84) = 0.006 694 379 9910
e^2 (NAD 27) = 0.006 768 658

NAD 27 = Clarke Spheroid of 1866
GRS 80 = NAD 83 reference ellipsoid

also

b = a(1 - f)

e^2 = f(2 - f) = (a^2 - b^2) / a^2

e^2 = (a^2 - b^2) / b^2

b. Cartesian to geodetic coordinate conversion. In the reverse case, given GRS 80 X, Y, Z coordinates, the conversion to NAD 83 geodetic coordinates (ϕ, λ, H) is performed using the following noniterative method (Soler and Hothem 1988):

$$\lambda = \arc\tan\frac{Y}{X} \tag{11-9}$$

The latitude ϕ and height h are computed using the following sequence. The initial reduced latitude β_o is first computed:

$$\tan\beta_0 = \frac{Z}{p}\left[(1 - f) + \frac{e^2 a}{r}\right] \tag{11-10}$$

where

$$p = \sqrt{X^2 + Y^2}$$
$$e^2 = 2f - f^2$$
$$r = \sqrt{p^2 + Z^2}$$

Directly solving for ϕ and h:

$$\tan\phi = \frac{Z(1 - f) + e^2 a\sin^3\beta_0}{(1 - f)(p - ae^2\cos^3\beta_0)} \tag{11-11}$$

$$h^2 = (p - a\cos\beta)^2 + (Z - b\sin\beta)^2 \tag{11-12}$$

where the final reduced latitude β is computed from

$$\tan\beta = (1 - f)\tan\phi \tag{11-13}$$

c. Transforms between other OCONUS datums may be performed by changing the ellipsoidal parameters *a*, *b*, and *f* to that datum's reference ellipsoid.

d. Example geocentric-geographic coordinate transform.

Geographic to geocentric (ϕ, λ, *h* to X, Y, Z) transform:

(1) Given any point:

ϕ_N = 35° 27' 15.217"

λ_W = 94° 49' 38.107"

λ = 360° - λ_W = 265.1727481°

h = 100 m (N = 0 assumed)

(2) Given constants (WGS 84):

a = 6,378,137 m b = a(1 - f) = 6,356,752.314

f = 1/298.257223563 e^2 = f(2 - f) = 6.694380 x 11^{-3}

$$R_N = a/(1 - e^2 \sin^2 \phi)^{1/2} = \underline{6,385,332,203}$$
$$X = (R_N + h) \cos\phi \cos\lambda = \underline{(-)437,710.553}$$
$$Y = (R_N + h) \cos\phi \sin\lambda = \underline{(-)5,182,990.319}$$
$$Z = \left(\frac{b^2}{a^2} R_N + H\right) \sin\phi = \underline{+3,679,090.327}$$

e. Geocentric (X, Y, Z) to geographic (ϕ, λ, H) transform.

Inversing the above X, Y, Z geocentric coordinates:

p = $(X^2 + Y^2)^{1/2}$ = 5,201,440.106

r = $(p^2 + Z^2)^{1/2}$ = 6,371,081.918

$$\beta_o = \tan^{-1} \frac{Z}{p}\left[(1 - f) + \frac{e^2 a}{r}\right] = 35.36295229°$$

$$\tan\phi = \frac{Z(1 - f) + e^2 a \sin^3 \beta_o}{(1 - f)(p - ae^2 \cos^3 \beta^o)}$$
$$= 0.712088398$$

ϕ = 35.45422693° = 35° 27' 15.217"

λ = $\tan^{-1}$(Y/X) = 85.17274810° (= 265.17274810°)

λ_W = 360° - λ = 94° 49' 38.107"

β = $\tan^{-1}$ [(1 - f) tan ϕ] = 35.36335663°

$$h^2 = (p - a\cos\beta)^2 + (Z - b\sin\beta)^2$$
$$= (81.458)^2 + (58.004)^2$$

h = 99.999 = 100 m

f. North American Datum of 1927 (Clarke Spheroid of 1866). Given a point with SPCS/Project coordinates on NAD 27, the point may be converted to X, Y, Z coordinates for use in subsequent adjustments.

ϕ_N = 35° 27' 15.217"

λ_W = 94° 49' 38.107" *h* or *H* = 100 m

(NAD 27 from SPCS X-Y ϕ,λ conversion using USACE program CORPSCON)

a = 6,378,206.4

b = 6,356,583.8

f = 1/294.978698

e^2 = 0.006768658
(NAD 27/Clarke 1866 Spheroid)

$$R_N = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}} = \underline{6,392,765.205}$$

$$X = (R_N + h) \cos\phi \cos\lambda = (-)\ 438,220.073 \text{ m}$$
$$Y = (R_N + h) \cos\phi \sin\lambda = (-)\ 5,189,023.612 \text{ m}$$
$$Z = \left(\frac{b^2}{a^2} R_N + H\right) \sin\phi = +3,733,466.852 \text{ m}$$

These geocentric coordinates (on NAD 27 reference) may be used to adjust subsequent GPS baseline vectors observed on WGS 84.

11-12. Rigorous Least Squares Adjustments of GPS Surveys

Adjustment of GPS networks on PC-based software is typically a trial-and-error process for both the free and constrained adjustments. When a least squares adjustment is performed on a network of GPS observations, the adjustment software will provide 2D or 3D coordinate accuracy estimates, variance-covariance matrix data for the adjusted coordinates, and related error ellipse data. Most software will provide relative accuracy estimates (length and azimuth) between points. Analyzing these various statistics is not easy, and they are also easily misinterpreted. Arbitrary rejection and readjustment in order to obtain a best fit (or best statistics) must be avoided. The original data reject criteria must be established and justified in a final report document.

a. When a series of loops are formed relative to a fixed point or off another loop, different redundant conditions are formed. (This is comparable to loops formed in conventional differential level nets.) These different loops allow forward baseline vector position computations to be made over different paths. From the different routes (loops) formed, different positional closures at a single fixed point result. These variances in position misclosures from the different routes provide additional data for assessing the internal consistency of the network, in addition to checking for blunders in the individual baselines. The number of different paths, or conditions, is partially related to the number of degrees of freedom in the network.

(1) Multiple observed baseline observations also provide additional redundancy or strength to a line or network since they are observed at two distinct times of varying satellite geometry and conditions. The amount of redundancy required is a function of the accuracy requirements of a particular survey.

(2) Performing a free adjustment on a complex network containing many redundancies is best performed using least squares methods. An example of such a network is shown in Figure 11-4. Approximate adjustment methods are difficult to evaluate when complex interweaving networks are involved.

(3) Baseline reduction vector component error statistics are usually carried down into the least squares adjustment; however, their use is not mandatory for lower order engineering surveys. GPS network least squares adjustments can be performed without all the covariance and correlation statistics from the baseline reduction.

(4) In practice, any station on the network can be held fixed for the free adjustment. The selected point is held fixed in all three coordinates, along with the orientation of the three axes and a network scale parameter. Usually one of the higher order points on the existing network is used.

b. Least squares adjustment software will output various statistics from the free adjustment to assist in detecting blunders and residual outliers in the free adjustment. Most commercial packages will display the normalized residual for each observation (GPS, EDM, angle, elevation, etc.), which is useful in detecting and rejecting residual outliers. The variance of unit weight is also important in evaluating the overall adequacy of the observed network. Other statistics, such as tau, chi-square, confidence levels, histograms, etc., are usually not significant for lower order USACE engineering projects, and become totally insignificant if one is not well versed in statistics and adjustment theory. Use of these statistics to reject data (or in reporting results of an adjustment) without a full understanding of their derivation and source within the network adjustment is ill-advised; they should be "turned off" if they are not fully understood.

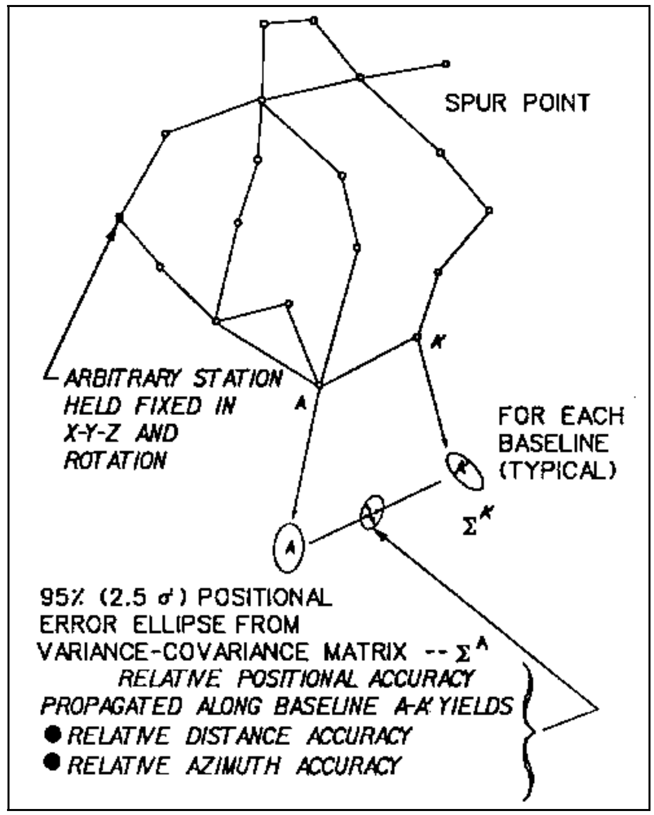

Figure 11-4. Free adjustment of a complex GPS network

c. Relative positional and distance accuracy estimates resulting from a free (unconstrained) geometric adjustment of a GPS network are usually excellent in comparison to conventional surveying methods. Loop misclosures and relative distance accuracies between points will commonly exceed 1:100,000.

d. Relative distance accuracy estimates between points in a network are determined by error propagation of the relative positional standard errors at each end of the line, as shown in Figure 11-4. Relative accuracy estimates may be derived for resultant distances or azimuths between the points. The relative distance accuracy estimates are those typically employed to assess the free (geometric) and constrained accuracy classifications, expressed as a ratio, such as 1:80,000. Since each point in the network will have its particular position variances, the relative distance accuracy propagated between any two points will also vary throughout the network.

(1) The minimum value (i.e., largest ratio) will govern the relative accuracy of the overall project. This minimum value (from a free adjustment) is then compared with the intended relative accuracy classification of the project to evaluate compliance. However, relative distance accuracy estimates should not be rigidly evaluated over short lines (i.e., less than 500 m).

(2) Depending on the size and complexity of the project, large variances in the propagated relative distance accuracies can result.

(3) When a constrained adjustment is performed, the adequacy of the external fixed stations will have a major impact on the resultant propagated distance accuracies, especially when connections are made to weak control systems. Properly weighted partially constrained adjustments will usually improve the propagated distance accuracies.

e. The primary criteria for assessing the adequacy of a particular GPS survey shall be based on the relative distance accuracy results from a minimally constrained free adjustment, not the fully constrained adjustment. This is due to the difficulty in assessing the adequacy of the surrounding network. Should the propagated relative accuracies fall below the specified level, then reobservation would be warranted.

(1) If the relative distance accuracies significantly degrade on a constrained adjustment (due to the inadequacy of the surrounding network), any additional connections to the network would represent a change in contract scope. A large variance of unit weight usually results in such cases.

(2) If only approximate adjustments are performed, then the relative distance accuracies may be estimated as a function of the loop or position misclosures, or the residual corrections to each observed length. For example, if a particular loop or line miscloses by 1 part in 200,000, then individual baseline relative accuracies can be assumed adequate if only a 1:20,000 survey is required.

f. Most commercial and Government adjustment software will output the residual corrections to each observed baseline (or actually baseline vector components). These residuals indicate the amount by which each segment was corrected in the adjustment. A least squares adjustment minimizes the sum of the squares of these baseline residual corrections.

(1) A number of commercial least squares adjustment software packages are available which will adjust GPS networks using standard IBM PC or PC-compatible computers. Those commonly used by USACE Commands include the following:

(a) TURBO-NET™, Geo-Comp, Inc., distributed by Geodetic Enterprises, Inc., PO Box 837, Odessa, FL 33556, (813) 920-4045.

(b) Geo-Lab™, distributed by GEOsurv, Inc., The Baxter Centre, 6-1050 Baxter Road, Ottawa, Ontario, Canada K2C 3P1, (613) 820-4545.

(c) FILLNET™, distributed by Ashtech, Inc., 1156-C Aster Avenue, Sunnyvale, CA, 94086, (408) 249-1314.

(d) ADJUST™, an adjustment program distributed by the National Geodetic Survey Information Center, Rockville, MD 20852.

(e) TRIMNET™, distruibuted by Trimble Navigation, Inc., 645 North Mary Avenue, P.O. Box 3642, Sunnyvale, CA, 94088-3642, (1-800-TRIMBLE).

(f) STAR*NET™, distributed by STARPLUS SOFTWARE, INC., 460 Boulevard Way, Oakland, CA, 94610, (510) 653-4836).

Annotated sample adjustment outputs from two commercial packages are shown in Figures 11-5 and 11-6.

(2) Relative GPS baseline standard errors can be obtained from the baseline reduction output and, in some software (i.e., Geo-Lab), can be directly input into the adjustment. These standard errors, along with their correlations, are given for each vector component (in X, Y, and Z). They are converted to relative weights in the adjustment. FILLNET allows direct input of vector component standard errors in a ±x + y ppm form. Correlations are not used in FILLNET. The following typical input (a priori) weighting is commonly used in FILLNET:

```
               ADJUSTMENT STATISTICS SUMMARY
               NETWORK = FTM1
               TIME = Wed Dec 15 18:13:40 1993


                           ADJUSTMENT SUMMARY

                 Network Reference Factor = 9.09
                 Chi-Square Test (α = 95%) = FAIL
                        Degrees of Freedom = 20.00


                      GPS OBSERVATIONS
                          Reference Factor = 9.09
                                         r = 20.00

GPS Solution    1 Reference Factor =    6.08   r =  2.38
GPS Solution    2 Reference Factor =   14.38   r =  2.66
GPS Solution    3 Reference Factor =    9.91   r =  2.06
GPS Solution    4 Reference Factor =    6.65   r =  2.21
GPS Solution    5 Reference Factor =   11.46   r =  2.13
GPS Solution    6 Reference Factor =    3.37   r =  1.96
GPS Solution    7 Reference Factor =    2.52   r =  2.05
GPS Solution    8 Reference Factor =    5.56   r =  2.10
GPS Solution    9 Reference Factor =   11.65   r =  2.45

                            WEIGHTING STRATEGIES:

     GPS OBSERVATIONS:
          No scalar weighting strategy was used

          No summation weighting strategy was used

          Station Error Strategy:
H.I. error = 0.0010
Tribrach error = 0.0010
```

Figure 11-5. TRIMNET sample adjustment output (Sheet 1 of 6)

COORDINATE ADJUSTMENT SUMMARY
NETWORK = FTM1
TIME = Wed Dec 15 18:13:41 1993

Datum = NAD-83
Coordinate System = Geographic
Zone = Global

Network Adjustment Constraints:
3 fixed coordinates in y
3 fixed coordinates in x
3 fixed coordinates in H
3 fixed coordinates in h

POINT NAME		OLD COORDS	ADJUST	NEW COORDS	1.96σ
1 C2PR					
	LAT=	40° 25′ 35.433030″	+0.000000″	40° 25′ 35.433030″	FIXED
	LON=	74° 20′ 41.879000″	+0.000000″	74° 20′ 41.879000″	FIXED
	ELL HT=	-29.8100m	+0.0000m	-29.8100m	FIXED
	ORTHO HT=	3.0400m	+0.0000m	3.0400m	FIXED
	GEOID HT=	-32.8500m	+0.0000m	-32.8500m	FIXED
2 FTM1					
	LAT=	40° 18′ 46.192066″	+0.000003″	40° 18′ 46.192069″	0.011390m
	LON=	74° 02′ 14.692854″	+0.000000″	74° 02′ 14.692854″	0.011542m
	ELL HT=	-22.3923m	-0.5752m	-22.9675m	0.016921m
	ORTHO HT=	0.0000m	+0.0000m	0.0000m	NOT KNOWN
3 MANT					
	LAT=	40° 02′ 18.425950″	+0.000000″	40° 02′ 18.425950″	FIXED
	LON=	74° 03′ 11.673310″	+0.000000″	74° 03′ 11.673310″	FIXED
	ELL HT=	-32.1600m	+0.0000m	-32.1600m	FIXED
	ORTHO HT=	1.1500m	+0.0000m	1.1500m	FIXED
	GEOID HT=	-33.3100m	+0.0000m	-33.3100m	FIXED
4 SIM3					
	LAT=	40° 28′ 06.064930″	+0.000000″	40° 28′ 06.064930″	FIXED
	LON=	74° 00′ 28.941590″	+0.000000″	74° 00′ 28.941590″	FIXED
	ELL HT=	-30.5800m	+0.0000m	-30.5800m	FIXED
	ORTHO HT=	2.1000m	+0.0000m	2.1000m	FIXED
	GEOID HT=	-32.6800m	+0.0000m	-32.6800m	FIXED

Ftm2
LAT 40° 18′ 46.05528″
LON 74° 02′ 14.77218″
EH -22.985 m

Figure 11-5. (Sheet 2 of 6)

```
              OBSERVATION ADJUSTMENT SUMMARY
              NETWORK = FTM1
              TIME = Wed Dec 15 18:13:43 1993



       OBSERVATION ADJUSTMENT (Tau = 2.85)

   GPS Parameter Group 1 GPS Observations
Azimuth rotation =  -0.2339 seconds          1.96σ = 0.0587 seconds
Deflection in latitude =  +0.0470 seconds    1.96σ = 0.1005 seconds
Deflection in longitude =  +0.5992 seconds   1.96σ = 0.1960 seconds
Network scale = 0.999995521210               1.96σ = 0.000000288587

OBS#  BLK#/  TYPE        BACKSIGHT/        UDVC/    OBSERVED/        1.96σ/     TAU
      REF#               INSTRUMENT/       UDPG/    ADJUSTED/        1.96σ/
                         FORESIGHT         SBNT     RESIDUAL         1.96σ

   1      1 gpsaz                   -**-   -**-    8°12'30.7755"   0.3399"    0.04
          1                         FTM1   -**-    8°12'30.7931"   0.1507"
                                    SIM3      1       +0.017599"   0.3046"

   2      1 gpsht                   -**-   -**-         -7.6308m   0.0495m    0.23
          1                         FTM1   -**-         -7.6160m   0.0219m
                                    SIM3      1       +0.014794m   0.0444m

   3      1 gpsds                   -**-   -**-      17448.3884m   0.0279m    0.34
          1                         FTM1   -**-      17448.4006m   0.0126m
                                    SIM3      1       +0.012253m   0.0249m

   4      2 gpsaz                   -**-   -**-  260°52'34.5985"   0.2105"    0.02
          1                         SIM3   -**-  260°52'34.5939"   0.0587"
                                    C2PR      1       -0.004628"   0.2022"

   5      2 gpsht                   -**-   -**-         +0.8693m   0.0696m    0.19
          1                         SIM3   -**-         +0.8516m   0.0282m
                                    C2PR      1       -0.017753m   0.0636m

   6      2 gpsds                   -**-   -**-      28957.8274m   0.0312m    0.84
          1                         SIM3   -**-      28957.8638m   0.0084m
                                    C2PR      1       +0.036478m   0.0300m

   7      3 gpsaz                   -**-   -**-    8°12'30.5976"   0.3100"    0.50
          1                         FTM1   -**-    8°12'30.7931"   0.1507"
                                    SIM3      1       +0.195536"   0.2710"

   8      3 gpsht                   -**-   -**-         -7.6089m   0.0316m    0.21
          1                         FTM1   -**-         -7.6160m   0.0219m
                                    SIM3      1       -0.007130m   0.0228m

   9      3 gpsds                   -**-   -**-      17448.4119m   0.0265m    0.33
          1                         FTM1   -**-      17448.4006m   0.0126m
                                    SIM3      1       -0.011284m   0.0233m

  10      4 gpsaz                   -**-   -**-  182°32'19.4314"   0.1863"    0.29
          1                         FTM1   -**-  182°32'19.3636"   0.0963"
                                    MANT      1       -0.067829"   0.1595"
```

Figure 11-5. (Sheet 3 of 6)

11	4 gpsht	-**-	-**-	-9.1984m	0.0388m	0.05
	1	FTM1	-**-	-9.1960m	0.0198m	
		MANT	1	+0.002485m	0.0333m	
12	4 gpsds	-**-	-**-	30496.2387m	0.0281m	0.33
	1	FTM1	-**-	30496.2272m	0.0143m	
		MANT	1	-0.011512m	0.0242m	
13	5 gpsaz	-**-	-**-	184°37'13.9839"	0.1196"	0.17
	1	SIM3	-**-	184°37'14.0097"	0.0586"	
		MANT	1	+0.025802"	0.1043"	
14	5 gpsht	-**-	-**-	-1.5901m	0.0399m	0.21
	1	SIM3	-**-	-1.5806m	0.0250m	
		MANT	1	+0.009521m	0.0311m	
15	5 gpsds	-**-	-**-	47890.3245m	0.0284m	0.68
	1	SIM3	-**-	47890.3000m	0.0138m	
		MANT	1	-0.024483m	0.0248m	
16	6 gpsaz	-**-	-**-	330°08'41.8933"	0.1115"	0.05
	1	MANT	-**-	330°08'41.8865"	0.0587"	
		C2PR	1	-0.006765"	0.0947"	
17	6 gpsht	-**-	-**-	+2.4222m	0.0348m	0.23
	1	MANT	-**-	+2.4307m	0.0242m	
		C2PR	1	+0.008509m	0.0250m	
18	6 gpsds	-**-	-**-	49729.7388m	0.0273m	0.07
	1	MANT	-**-	49729.7364m	0.0144m	
		C2PR	1	-0.002441m	0.0232m	
19	7 gpsaz	-**-	-**-	2°31'42.5754"	0.1792"	0.14
	1	MANT	-**-	2°31'42.6064"	0.0960"	
		FTM1	1	+0.031010"	0.1513"	
20	7 gpsht	-**-	-**-	+9.1917m	0.0324m	0.09
	1	MANT	-**-	+9.1951m	0.0198m	
		FTM1	1	+0.003409m	0.0256m	
21	7 gpsds	-**-	-**-	30496.2264m	0.0268m	0.02
	1	MANT	-**-	30496.2272m	0.0143m	
		FTM1	1	+0.000753m	0.0226m	
22	8 gpsaz	-**-	-**-	295°53'30.6165"	0.1904"	0.21
	1	FTM1	-**-	295°53'30.5663"	0.0923"	
		C2PR	1	-0.050255"	0.1665"	
23	8 gpsht	-**-	-**-	-6.7766m	0.0341m	0.33
	1	FTM1	-**-	-6.7642m	0.0224m	
		C2PR	1	+0.012405m	0.0257m	
24	8 gpsds	-**-	-**-	29010.8695m	0.0272m	0.05
	1	FTM1	-**-	29010.8678m	0.0129m	
		C2PR	1	-0.001702m	0.0239m	
25	9 gpsaz	-**-	-**-	295°53'30.6764"	0.2227"	0.37
	1	FTM1	-**-	295°53'30.5663"	0.0923"	
		C2PR	1	-0.110138"	0.2026"	

Figure 11-5. (Sheet 4 of 6)

26	9 gpsht	-**-	-**-	-6.7600m	0.0564m	0.06
	1	FTM1	-**-	-6.7642m	0.0224m	
		C2PR	1	-0.004171m	0.0518m	
27	9 gpsds	-**-	-**-	29010.8399m	0.0319m	0.66
	1	FTM1	-**-	29010.8678m	0.0129m	
		C2PR	1	+0.027863m	0.0291m	

Figure 11-5. (Sheet 5 of 6)

SUMMARY OF COVARIANCES
NETWORK = FTM1
TIME = Wed Dec 15 18:13:45 1993

FROM/ TO	AZIMUTH/ DELTA H	1.96σ 1.96σ	DISTANCE/ DELTA h	1.96σ 1.96σ	HOR PREC
C2PR FTM1	115°41'34" +6.8425m	0.08" 0.0169m	29010.998m -**-	0.0114m -**-	1: 2536313
C2PR MANT	-**- -**-	-**- -**-	-**- -**-	-**- -**-	-**-
C2PR SIM3	-**- -**-	-**- -**-	-**- -**-	-**- -**-	-**-
FTM1 MANT	182°32'20" -9.1925m	0.08" 0.0169m	30496.364m -**-	0.0114m -**-	1: 2675331
FTM1 SIM3	8°12'31" -7.6125m	0.14" 0.0169m	17448.479m -**-	0.0114m -**-	1: 1527827
MANT SIM3	-**- -**-	-**- -**-	-**- -**-	-**- -**-	-**-

Figure 11-5. (Sheet 6 of 6)

```
PROGRAM FILLNET, Version 3.0.00
LICENSED TO: ASHTECH INC.


Fillnet Input File  acts                         40.3    74.1

a =  6378137.000    1/f =  298.2572221   W      Longitude positive WEST

PRELIMINARY COORDINATES:
                      LAT.              LON.          ELEV.     G.H.   CONSTR.

  1         FTM2 40 18 46.25804    74  2 14.14056    47.648    0.000
  2   FFF   MANT 40  2 18.42595    74  3 11.67331   -32.160    0.000
  3   FFF   C2PR 40 25 35.43303    74 20 41.87900   -29.810    0.000
  4   FFF   SIM3 40 28  6.06493    74  0 28.94159   -30.580    0.000
  5         FTM1 40 18 46.19156    74  2 14.69409   -24.152    0.000

GROUP 1, NO. OF VECTORS AND BIAS CONSTRAINTS:

 11      0.000  0.000    0.000  0.000    0.000  0.000    0.000  0.000

VECTORS:
                 DX          DY          DZ       LENGTH    ERROR CODES

FTM1   FTM2      -1.061      -3.124      -3.223       4.612  3 51.0   51.0  2
FTM1   FTM2      -1.059      -3.126      -3.223       4.613  3 51.0   51.0  2
C2PR   MANT   31466.066 -20017.367 -32897.190  49729.603  3 51.0   51.0  3
C2PR   FTM1   27361.521    -755.791   -9612.798  29010.859  3 51.0   51.0  3
MANT   SIM3   -4788.603  30711.771  36432.475  47890.175  3 51.0   51.0  3
MANT   FTM1   -4104.542  19261.593  23284.378  30496.204  3 51.0   51.0  3
SIM3   FTM1     684.063 -11450.182 -13148.091  17448.407  3 51.0   51.0  3
MANT   FTM1   -4104.553  19261.590  23284.377  30496.203  3 51.0   51.0  3
C2PR   SIM3   26677.446  10694.407   3535.282  28957.809  3 51.0   51.0  3
C2PR   FTM1   27361.503    -755.790   -9612.797  29010.842  3 51.0   51.0  3
SIM3   FTM1     684.055 -11450.181 -13148.088  17448.404  3 51.0   51.0  3

 SHIFTS:
   1  -6.254 -14.914 -70.647
   2   0.000   0.000   0.000
   3   0.000   0.000   0.000
   4   0.000   0.000   0.000
   5   0.009   0.035   1.170

ADJUSTED VECTORS, GROUP 1:
                             DX,DY,DZ    V        DN,DE,DU    v       v'

FTM1   FTM2     3213B          -1.060  0.001        -4.214 -0.001   -0.3
                               -3.125 -0.001        -1.876  0.001    0.1
                               -3.223 -0.000        -0.014  0.001    0.2

FTM1   FTM2     3213A          -1.060 -0.001        -4.214  0.001    0.3
                               -3.125  0.001        -1.876 -0.001   -0.1
                               -3.223 -0.000        -0.014 -0.001   -0.2

C2PR   MANT     3203C       31466.069 -0.002    -43116.924  0.019    0.3
                           -20017.389  0.005     24778.270 -0.000   -0.0
                           -32897.156  0.021       -20.476  0.009    0.2

C2PR   FTM1     3203C       27361.531  0.000    -12649.806  0.009    0.2
                             -755.810  0.013     26107.654  0.004    0.1
```

Figure 11-6. FILLNET sample adjustment output (Sheet 1 of 3)

```
                                 -9612.775  0.001      53.853 -0.009   -0.2

MANT    SIM3      3203B          -4788.587  0.006    47738.483 -0.015  -0.2
                                 30711.758 -0.003     3808.387  0.005   0.1
                                 36432.476 -0.015       36.869 -0.006  -0.1

MANT    FTM1      3203B          -4104.538 -0.001    30467.118 -0.010  -0.3
                                 19261.579 -0.009     1329.384 -0.003  -0.1
                                 23284.381 -0.006       74.329  0.003   0.1

SIM3    FTM1      3203B            684.050 -0.009   -17271.365  0.003   0.1
                                -11450.179 -0.002    -2479.003 -0.009  -0.4
                                -13148.095  0.003       37.460  0.001   0.1

MANT    FTM1      3203C          -4104.538  0.010    30467.118 -0.009  -0.2
                                 19261.579 -0.006     1329.384  0.008   0.2
                                 23284.381 -0.005       74.329  0.003   0.1

C2PR    SIM3      3203A          26677.482  0.021     4621.559  0.002   0.1
                                 10694.369 -0.001    28586.657  0.020   0.6
                                  3535.320  0.009       16.393  0.011   0.3

C2PR    FTM1      3203A          27361.531  0.018   -12649.806  0.004   0.1
                                  -755.810  0.012    26107.654  0.021   0.6
                                 -9612.775 -0.000       53.853 -0.005  -0.1

SIM3    FTM1      3203A            684.050 -0.001   -17271.365 -0.001  -0.1
                                -11450.179 -0.003    -2479.003 -0.002  -0.1
                                -13148.095  0.000       37.460  0.002   0.1

S.E. OF UNIT WEIGHT =      0.278

NUMBER OF -
  OBS. EQUATIONS                33
  UNKNOWNS                      10
  DEGREES OF FREEDOM            23
  ITERATIONS                     0


GROUP  1 ROT. ANGLES (sec.) AND SCALE DIFF. (ppm):

HOR. SYSTEM           0.342   -0.057    0.026    0.101
STD. ERRORS           0.044    0.030    0.025    0.120
XYZ SYSTEM           -0.110    0.178    0.277

ADJUSTED POSITIONS:

                         LAT.              LON.            ELEV.    STD. ERRORS (m)

  1  FTM2     40 18 46.05528   74  2 14.77217    -22.999 0.003 0.004 0.004
  2  MANT     40  2 18.42595   74  3 11.67331    -32.160 0.000 0.000 0.000
  3  C2PR     40 25 35.43303   74 20 41.87900    -29.810 0.000 0.000 0.000
  4  SIM3     40 28  6.06493   74  0 28.94159    -30.580 0.000 0.000 0.000
  5  FTM1     40 18 46.19186   74  2 14.69262    -22.982 0.003 0.003 0.004

ACCURACIES (m):
                                      D. LAT.     D. LON.      VERT.

 FTM1            FTM2                  0.001       0.001       0.001
 FTM1            FTM2                  0.001       0.001       0.001
```

Figure 11-6. (Sheet 2 of 3)

C2PR	MANT	0.000	0.000	0.000
C2PR	FTM1	0.003	0.003	0.004
MANT	SIM3	0.000	0.000	0.000
MANT	FTM1	0.003	0.003	0.004
SIM3	FTM1	0.003	0.003	0.004
MANT	FTM1	0.003	0.003	0.004
C2PR	SIM3	0.000	0.000	0.000
C2PR	FTM1	0.003	0.003	0.004
SIM3	FTM1	0.003	0.003	0.004

```
*****************************************************************
****                                                         ****
****                 ESTIMATES OF PRECISION                  ****
****                                                         ****
****      Based on the VECTOR ACCURACIES produced by         ****
****                       FILLNET                           ****
****                                                         ****
****    This is a reasonable estimate of the accuracies      ****
****      of the vectors in the network at 1 SIGMA.          ****
****                                                         ****
*****************************************************************
```

VECTOR		LENGTH	PPM(h)	RATIO(h)	PPM(v)	RATIO(v)
FTM1	FTM2	4.613	306.6	1: 3262	216.8	1: 4613
FTM1	FTM2	4.613	306.6	1: 3262	216.8	1: 4613
C2PR	MANT	49729.591	0.0	1: 0	0.0	1: 0
C2PR	FTM1	29010.862	0.1	1: 6837914	0.1	1: 7252715
MANT	SIM3	47890.165	0.0	1: 0	0.0	1: 0
MANT	FTM1	30496.198	0.1	1: 7188001	0.1	1: 7624049
SIM3	FTM1	17448.407	0.2	1: 4112620	0.2	1: 4362102
MANT	FTM1	30496.198	0.1	1: 7188001	0.1	1: 7624049
C2PR	SIM3	28957.832	0.0	1: 0	0.0	1: 0
C2PR	FTM1	29010.862	0.1	1: 6837914	0.1	1: 7252715
SIM3	FTM1	17448.407	0.2	1: 4112620	0.2	1: 4362102
SIM3	FTM1	17448.407	0.2	1: 4112620	0.2	1: 4362102

Figure 11-6. (Sheet 3 of 3)

(a) Fixed: ±3 mm (Lat) ± 5 mm (Long) + 1 ppm ±5 mm (Height) + 1 ppm

(b) Float: ± 6 mm (Lat) ± 10 mm (Long) + 2 ppm ± 10 mm (Height) + 2 ppm

The optimum standard errors shown have been found to be reasonable in standard USACE work where extremely long baselines are not involved. Use of these optimum values is recommended for the first adjustment iteration.

(3) The adequacy of the initial network weighting described in (2) above is indicated by the variance of unit weight (or variance factor in Geo-Lab) which equals the square of the standard error of unit weight (FILLNET). The variance of unit weight should range between 0.5 and 1.5 (or the standard error of unit weight should range between 0.7 and 1.2), with an optimum value of 1.0 signifying realistic weighting of the GPS input observations. A large unit variance (say 5.0) indicates the initial GPS standard errors were too optimistic (low). A low unit variance (say 0.1) indicates the results from the adjustment were better than the assumed GPS baseline precisions used. This unit variance test, however, is generally valid only when a statistically significant number of observations are involved. This is a function of the number of degrees of freedom shown on the adjustment. To evaluate the adequacy of the unit weight, a test such as chi-square in Geo-Lab is performed. Failure of such a test indicates the variance factor statistic may not be statistically valid, including any rejections made using this value.

(4) The input standard errors can easily be juggled in order to obtain a variance of unit weight near 1.0. This trial-and-error method is generally not a good practice. If the input weights are changed, they should not be modified beyond reasonable levels (e.g., do not input a GPS standard error of ±50 + 50 ppm in order to get a good unit variance). If input standard errors are modified, these modifications should be the same for all lines, not just selected ones. Any such modifications of a priori standard errors must be justified in the adjustment report.

(5) Changing the magnitude of the input standard errors/weights will not change the adjusted position or residual results in a free adjustment provided all weight changes are made equally. Although the reference variance will change, the resultant precisions (relative line accuracies) will not change. (This is not true in a constrained adjustment.) Therefore, the internal accuracy of a survey can be assessed based on the free adjustment line accuracies regardless of the initial weighting or variance of unit weight.

(6) The magnitude of the residual corrections shown in the sample adjustments may be assessed by looking for blunders or outliers; however, this assessment should be performed in conjunction with the related normalized residual (FILLNET) or standardized residual (Geo-Lab) statistic. This statistic is obtained by multiplying the residual by the square root of the input weight (the inverse of the square of the standard error). If the observations are properly weighted, the normalized residuals should be around 1.0. Most adjustment software will flag normalized residuals which exceed selected statistical outlier tests. Such flagged normalized residuals are candidates for rejection. A rule-of-thumb reject criterion should be set at three times the standard error of unit weight, again provided that the standard error of unit weight is within the acceptable range given in (3) above. All rejected GPS observations must be justified in the adjustment report clearly describing the test used to remove the observation from the file.

(7) Error ellipses, or 3D error ellipsoids, generated from the adjustment variance-covariance matrices for each adjusted point in Geo-Lab are also useful in depicting the relative positional accuracy. The scale of the ellipse may be varied as a function of the 2D deviation. Usually a 2.45-σ, or 95 percent, probability ellipse is selected for output. The size of the error ellipse will give an indication of positional reliability, and the critical relative distance/azimuth accuracy estimate between two adjacent points is a direct function of the size of these positional ellipses.

(8) The relative distance accuracy estimates (i.e., relative station confidence limits in Geo-Lab and estimates of precision in FILLNET) are used to evaluate acceptability of a survey. This is done using a free adjustment. The output is shown as a ratio (FILLNET) or in parts per million (Geo-Lab). Note that FILLNET uses a 1-σ line accuracy. The resultant ratios must be divided by 2 in order to equate them to FGCS 95 percent criteria. Geo-Lab is set to default to the 95 percent level.

(9) Further details on these statistical evaluations are beyond the scope of this manual. Technical references listed under paragraph A-1 should be consulted.

g. The following is a summary of a network adjustment sequence recommended by the NGS for surveys which are connected with the NGRS:

(1) A minimally constrained 3D adjustment is done initially as a tool to validate the data, check for blunders and systematic errors, and to look at the internal consistency of the network.

(2) A 3D horizontal constrained adjustment is performed holding all previously published horizontal control points fixed and one height constraint. If the fit is poor, then a readjustment is considered. All previous observations determining the readjusted stations are considered in the adjustment.

(3) A fully constrained vertical adjustment is made to determine the orthometric heights. All previously published benchmark elevations are held fixed along with one horizontal position in a 3D adjustment. Geoid heights are predicted using the latest model.

(4) A final free adjustment is performed in which relative accuracy estimates are computed.

11-13. Evaluation of Adjustment Results

A survey shall be classified based on its horizontal point closure ratio, as indicated in Table 11-1 or the vertical elevation difference closure standard given in Table 11-2.

Table 11-1
USACE Point Closure Standards for Horizontal Control Surveys

USACE Classification	Point Closure Standard (Ratio)
Second Order Class I	1:50,000
Second Order Class II	1:20,000
Third Order Class I	1:10,000
Third Order Class II	1: 5,000
4th Order - Construction Layout	1: 2,500 - 1:20:000

Table 11-2
USACE Point Closure Standards for Vertical Control Surveys

USACE Classification	Point Closure Standard (mm)
Second Order Class I	$6mm\sqrt{K}$
Second Order Class II	$8mm\sqrt{K}$
Third Order	$12mm\sqrt{K}$
4th Order - Construction Layout	$24mm\sqrt{K}$

($\sqrt{K}$ is square root of distance K in kilometers)

a. Horizontal control standards. The horizontal point closure is determined by dividing the linear distance misclosure of the survey into the overall circuit length of a traverse, loop, or network line/circuit. When independent directions or angles are observed, as on a conventional survey (i.e., traverse, trilateration, or triangulation), these angular misclosures may optionally be distributed before assessing positional misclosure. In cases where GPS vectors are measured in geocentric coordinates, then the 3D positional misclosure is assessed.

(1) Approximate surveying. Approximate surveying work should be classified based on the survey's estimated or observed positional errors. This would include absolute GPS and some differential GPS techniques with positional accuracies ranging from 10 to 150 ft (2DRMS). There is no order classification for such approximate work.

(2) Higher order surveys. Requirements for relative line accuracies exceeding 1:50,000 are rare for most USACE applications. Surveys requiring accuracies of First-Order (1:100,000) or better should be performed using FGCS standards and specifications, and must be adjusted by the NGS.

(3) Construction layout or grade control (Fourth-Order). This classification is intended to cover temporary control used for alignment, grading, and measurement of various types of construction, and some local site plan topographic mapping or photo mapping control work. Accuracy standards will vary with the type of construction. Lower accuracies (1:2,500 - 1:5,000) are acceptable for earthwork, dredging, embankment, beach fill, and levee alignment stakeout and grading, and some site plan, curb and gutter, utility building foundation, sidewalk, and small roadway stakeout. Moderate accuracies (1:5,000) are used in most pipeline, sewer, culvert, catch basin, and manhole stakeout, and for general residential building foundation and footing construction, major highway pavement, and concrete runway stakeout work. Somewhat higher accuracies (1:10,000 - 1:20,000) are used for aligning longer bridge spans, tunnels, and large commercial structures. For extensive bridge or tunnel projects, 1:50,000 or even 1:100,000 relative accuracy alignment work may be required. Vertical grade is usually observed to the nearest 0.005 m for most construction work, although 0.04-m accuracy is sufficient for riprap placement, grading, and small-diameter-pipe placement. Construction control points are typically marked by semi-permanent or temporary monuments (e.g., plastic hubs, P-K nails, wooden grade stakes). Control may be established by short, nonredundant spur shots, using total stations or GPS, or by single traverse runs between two existing permanent control points. Positional accuracy

will be commensurate with, and relative to, that of the existing point(s) from which the new point is established.

b. Vertical control standards. The vertical accuracy of a survey is determined by the elevation misclosure within a level section or level loop. For conventional differential or trigonometric leveling, section or loop misclosures (in millimeters) shall not exceed the limits shown in Table 11-2, where the line or circuit length (K) is measured in kilometers. Fourth-Order accuracies are intended for construction layout grading work. Procedural specifications or restrictions pertaining to vertical control surveying methods or equipment should not be over-restrictive.

11-14. Final Adjustment Reports and Submittals

a. A variety of free and/or constrained adjustment combinations may be specified for a contracted GPS survey. Specific stations to be held fixed may be indicated or a contractor may be instructed to determine the optimum adjustment, including appropriate weighting for constrained points. When fixed stations are to be partially constrained, then appropriate statistical information must be provided--either variance-covariance matrices or relative positional accuracy estimates which may be converted into approximate variance-covariance matrices in the constrained adjustment. All rejected observations will be clearly indicated, along with the criteria/reason used in the rejection.

b. When different combinations of constrained adjustments are performed due to indications of one or more fixed stations causing undue biasing of the data, an analysis shall be made as to a recommended solution which provides the best fit for the network. Any fixed control points which should be readjusted to anomalies from the adjustment(s) should be clearly indicated in a final analysis recommendation.

c. The final adjusted horizontal and/or vertical coordinate values shall be assigned an accuracy classification based on the adjustment statistical results. This classification shall include both the resultant geodetic/Cartesian coordinates and the baseline differential results. The final adjusted coordinates shall state the 95 percent confidence region of each point and the accuracy in parts per million between all points in the network. The datum and/or SPCS will be clearly identified for all coordinate listings.

d. Final report coordinate listings may be required on hard copy as well as on a specified computer media.

e. It is recommended that a scaled plot be submitted with the adjustment report showing the proper locations and designations of all stations established.